Appleman and Tree

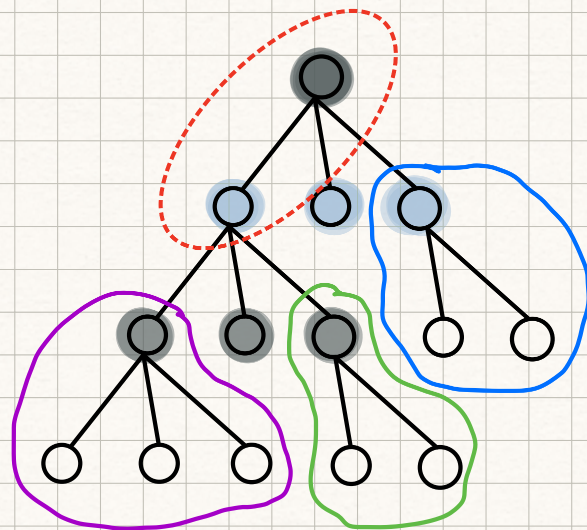
Tree → n vertices

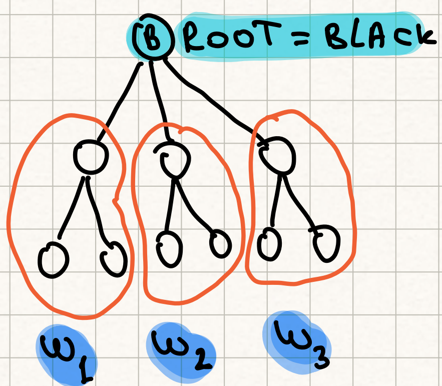
k + 1 parts

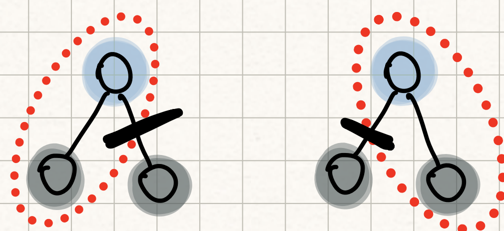
dp(v, b)

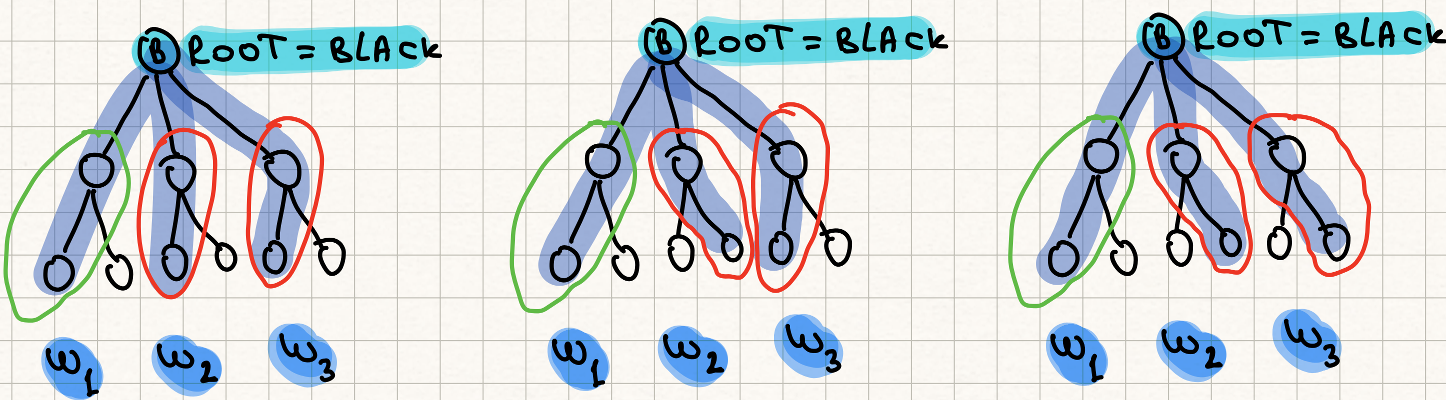
dp(v, w)

B W









$\boxed{w_1 \times w_2 \times w_3}$ Different ways

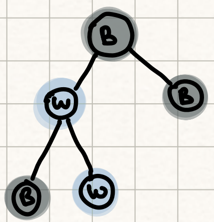
dp_black(u): ways to break subtree rooted at u

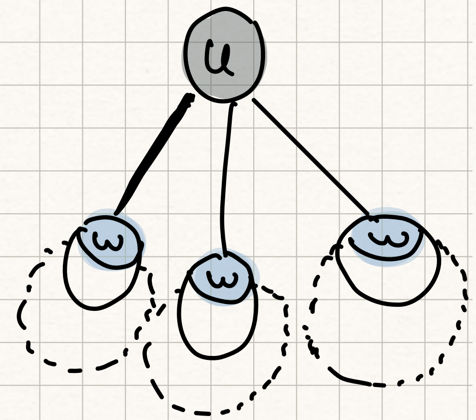
dp_WHITE(u): component containing u has all WHITE and REST 1 black

(u) → black

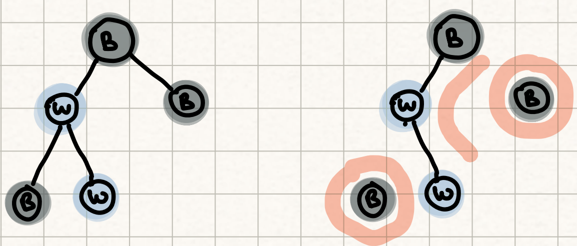
$$dp.black(u) = \prod_{c_i} dp\_black(c_i) + \prod_{c_i} dp\_WHITE(c_i)$$

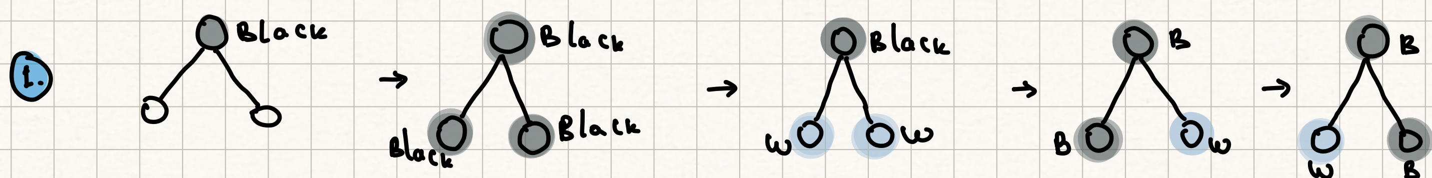
$$\rightarrow \boxed{\prod_{c_i} \left(dp\_b(c_i) + dp\_w(c_i) \right)}$$

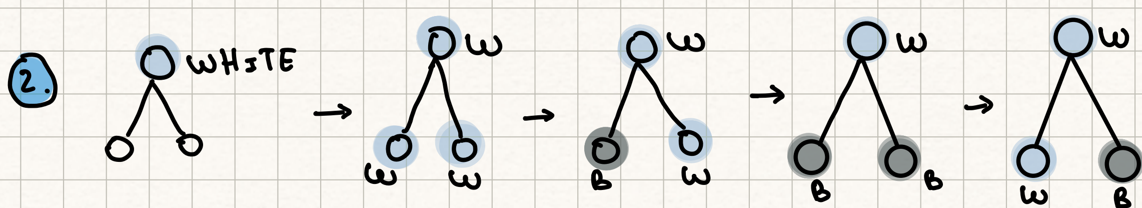


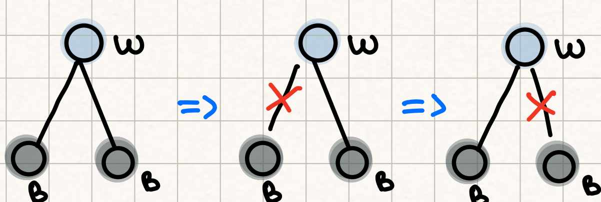


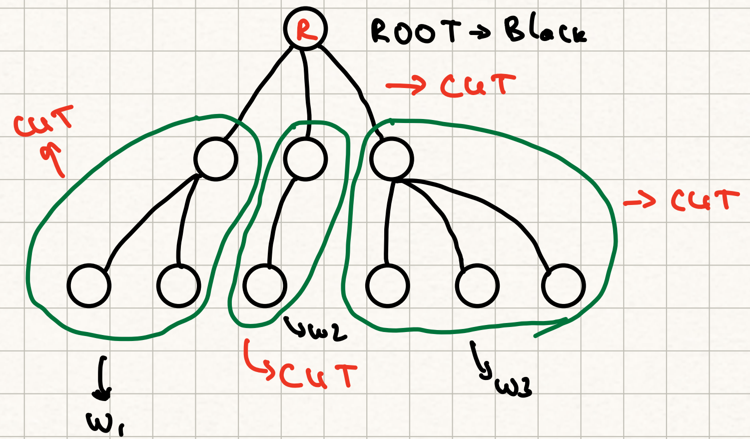
$dp\_WHITE(c_i)$





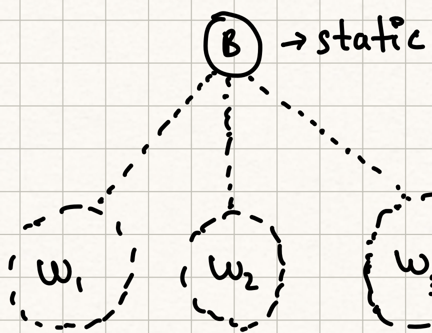


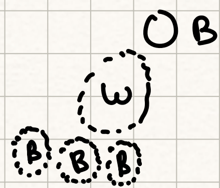




(*) same subproblems for subtrees (children)

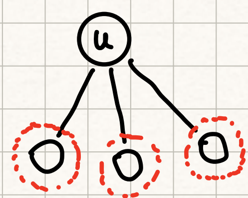
(*) $w_1 \times w_2 \times w_3$ ways





dp_black(u)

↳ ways to break subtree rooted at u to Black(s)



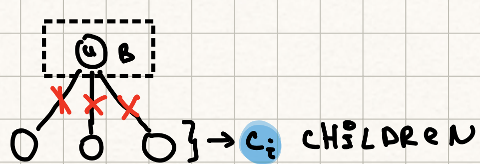
dp_white(u)

↳ component containing u has all white and rest comps have 1 black

$dp\_black(u) = \prod_{c_i} dp\_black(c_i) + \prod_{c_i} dp\_white(c_i) =$

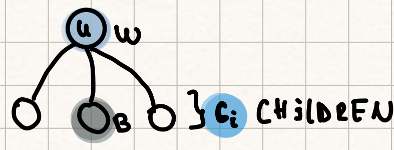
u → black

$= \prod_{c_i} (dp\_black(c_i) + dp\_white(c_i))$

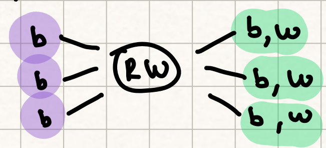


dp_black(u)

u → white



$dp\_black(c_2) \rightarrow$ ROOT_W

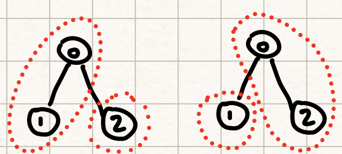


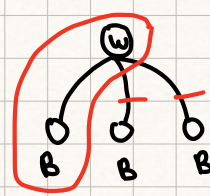
$$dp\_black(c_1) \times [\, dp\_black(c_2) + dp\_white(c_2)\,] \times [\, dp\_black(c_3) + dp\_white(c_3)\,]$$

$$dp\_black(u) = dp.b(c_1) \times (dp\_b(c_2) + dp.w(c_2)) \times \ldots (dp\_b(c_n) + dp.w(c_n)) =$$

$$= \sum \underbrace{LW(i-1)}_{\text{prefix}}\, dp\_b(i)\, \underbrace{RW(i+1)}_{\text{suffix}}$$

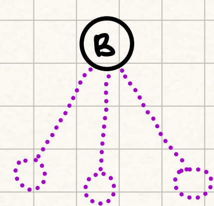
$$dp\_white(u) = LW(i)$$





IN DP ON TREE IT'S good to analyse the cases.

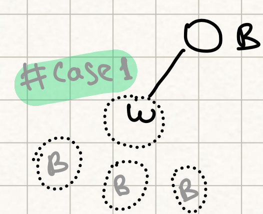
We always need to find a solution entire subtree.



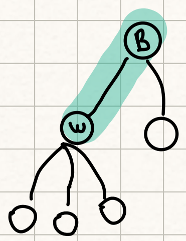
dp[node][0]

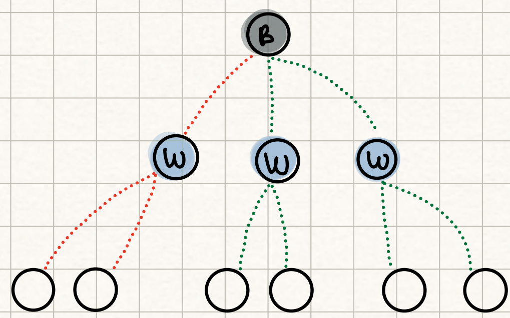
dp[node][1]

dp[node][0] → subtree rooted at node to have no black vertex

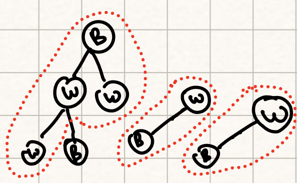


dp[node][1] → subtree rooted at node has exactly one black vertex.



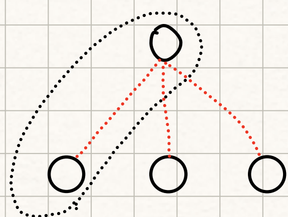


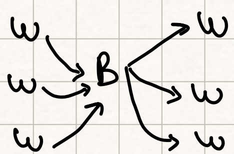
If root is Black,
there are two options:
ⓐ Connect subtree_WHITE
ⓑ Disconnect subtrees (edges)

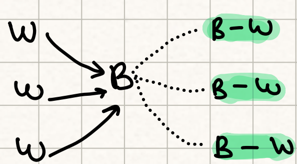


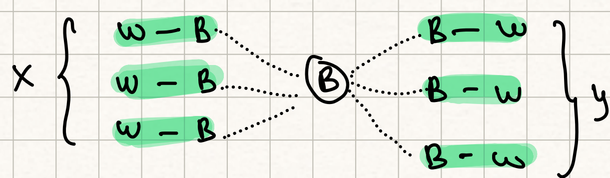
dp_black(u) → blacks

dp_white(u) → all white, others black

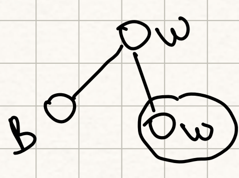






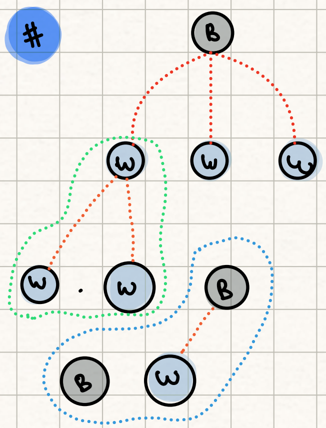


X * y Different ways



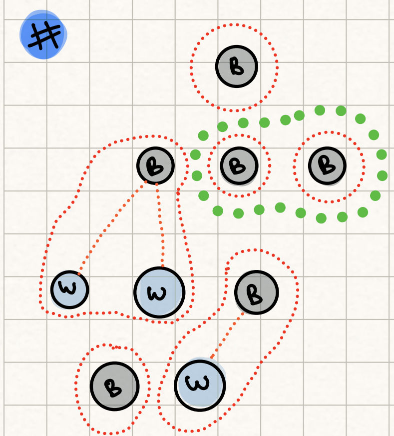
dp[w][w] = 1
dp[w][B] = 0
dp[B][w] = 0
dp[B][B] = 1

Different cases:



if Root is Black children could be only-WHITE components

OR



children are separated one-block vertex components.

Total way of ① subtree => $dp\_black(c_i) + dp\_white(c_i)$

$dp\_black(u) = \prod_{c_i} dp\_black(c_i) + \prod_{c_i} dp\_white(c_i)$ (u → black)

$dp\_white(u) = 0$ if ⓤ is black

$dp\_black(u) = \prod_{c_i} dp\_black(c_i)$ (u → white)